Journal of Guidance, Control, and Dynamics
Vol. 32, No. 5, September–October 2009

Engineering Notes

Time-Optimal Detumbling Control of Spacecraft

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DOI: 10.2514/1.43189

I. Introduction

THE problem of designing fast detumbling maneuvers for a spacecraft with restricted actuation torques arises in many space applications. Satellites are often equipped with an attitude control system (ACS), which usually has two modes of operation: detumbling and stabilization. In detumbling mode, the ACS controller is responsible for dumping the initial angular velocity from when the satellite was in the idle mode; this situation occurs when the satellite is released from the launch vehicle or when power is so low that ACS has to be turned off to save power [1]. Only after this phase can the stabilization mode be activated to control the orientation of the satellite to align antennas toward the Earth. Reaction wheels are commonly used as the actuators of the ACSs. Satellites frequently need fast maneuvers [2], minimum time maneuvers, while the speed is restricted owing to a low level capacity of the actuators. Hence, planning optimal maneuvers is highly desired.

Alternatively, spin-stabilized spacecraft allow simple attitude maneuvers without the need for complex control systems. The spacecraft can be spun up around its axisymmetric axis to stabilize the orientation of the vehicle axis through the gyroscopic effect. This method is also widely used to stabilize the final stage of a launch vehicle [3]. However, when a spacecraft is in the tumbling motion, its angular velocity is not parallel to the axisymmetric axis. Therefore, the objective in deployment of a spin-stabilized spacecraft [4] is to bring the spacecraft form the tumbling motion to the state wherein the spacecraft spins around a single axis.

The detumbling or passivation of a satellite is also required before on-orbit servicing of the satellite or its retrieval [5]. For such a mission, an orbital maneuvering vehicle can be used to apply torques to the target satellite for removing any relative velocity [5,6]. Also, the vehicle can be equipped by an articulating arm with a grappling device on it that could be driven to capture a tumbling target satellite and then detumble it [7,8]. After capture of an uncontrolled tumbling satellite by a space manipulator, the satellite should be brought to rest in minimum time. Again, the restriction of the manipulator end effector to provide "braking torques" for the fast maneuvers motivates an optimal trajectory planning.

Optimal detumbling of multibody systems has been considered for spacecraft possessing appendages, such as a robotic manipulator, with well-controlled motion relative to the spacecraft to achieve detumbling [9–13]. However, because of the complexity of dynamics of space manipulators, only a numerical solution or intensive trial-and-error procedures have been found for the optimization problem. The problem of time-optimal detumbling control of rigid spacecraft is formulated as a nonlinear programming and solved numerically by using an iterative procedure in [14], while nonoptimal control approaches have been reported in [15–17]. There also exists a fair amount of research done on the time-optimal reorientation control, rest to rest, of rigid spacecraft [18–20] and a survey, for example, can be found in [21].

This paper presents a closed-form solution for time-optimal detumbling control of a rigid spacecraft with the constraint on maintaining the Euclidean norm of the braking torques below a prescribed value. The final angular rate can be specified as zero or any vector parallel to the eigenaxis. The optimal control theory and Pontryagin's principle are applied to derive the optimal solution, which is not only easy to implement but also gives a great deal of insight. First, it will be shown that for our particular optimal control problem, the system costates and states are related by a nonlinear but static function. Subsequently, the optimal control law is explicitly derived in the form of a nonlinear state feedback. The magnitude of the system angular momentum is found to be linearly deceasing with time, leading to a simple expression for the terminal time needed for control implementation. Furthermore, the time-optimal controller is extended for the case of nonzero terminal velocity. To this end, the time-optimal control technique is applied to derive a nonlinear feedback control law which can drive a tumbling axisymmertic spacecraft to a final spin-stabilized state. Finally, the time-optimal detumbling technique is illustrated through numerical examples.

II. Time-Optimal Control

A. Passivation of Tumbling Spacecraft

Dynamics of the rotational motion of a rigid body (spacecraft) can be expressed by Euler's equation as

$$\dot{\boldsymbol{\omega}} = \boldsymbol{\phi}(\boldsymbol{\omega}) + \mathbf{I}_c^{-1}\boldsymbol{\tau} \tag{1}$$

where $\boldsymbol{\omega}$ and $\boldsymbol{\tau}$ denote the vectors of the angular velocity and input torque, both of which are expressed in the fixed-body frame, $\mathbf{I}_c$ is the inertia tensor, and

$$\boldsymbol{\phi}\,(\boldsymbol{\omega}) = -\mathbf{I}_c^{-1}(\boldsymbol{\omega} \times \mathbf{I}_c\boldsymbol{\omega}) \tag{2}$$

The time-optimal control problem being considered here is how to drive the spacecraft from the given initial angular velocity $\boldsymbol{\omega}(0)$ to rest in *minimum time* while the Euclidean norm of the torque input is restricted to be below a prescribed value $\tau_{\max}$. That is the following cost function:

$$J = \int_0^{t_f} 1\mathrm{d}t$$

is minimized subject to the terminal condition $\boldsymbol{\omega}(t_f) = 0$ while the input torque trajectory should satisfy

$$\|\boldsymbol{\tau}\| \le \tau_{\max} \tag{3}$$

where $\|\cdot\|$ denotes the L_2 norm or the Euclidean norm of a vector. Denoting vector $\boldsymbol{\lambda} \in \mathbb{R}^3$ as the costates, we can write the system Hamiltonian as

$$H = 1 + \boldsymbol{\lambda}^T\boldsymbol{\phi}(\boldsymbol{\omega}) + (\mathbf{I}_c^{-1}\boldsymbol{\lambda})^T\boldsymbol{\tau} \tag{4}$$

Then, the theory of optimal control [22] dictates that the time derivative of the costates must satisfy

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Presented as Paper 7274 at the 2008 AIAA Guidance, Navigation and Control Conference and Exhibit, Honolulu, HI, 18–21 August 2008; received 12 January 2009; accepted for publication 6 June 2009.

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$$\dot{\boldsymbol{\lambda}} = -\frac{\partial H}{\partial \boldsymbol{\omega}} = -\frac{\partial \boldsymbol{\phi}^T}{\partial \boldsymbol{\omega}}\boldsymbol{\lambda} \tag{5}$$

where

$$\frac{\partial \boldsymbol{\phi}^T}{\partial \boldsymbol{\omega}} = \mathbf{I}_c[\boldsymbol{\omega}\times]\mathbf{I}_c^{-1} - [\mathbf{I}_c\boldsymbol{\omega}\times]\mathbf{I}_c^{-1} \tag{6}$$

and skew-symmetric matrix $[\mathbf{a}\times]$ represents the *cross product*, that is, $[\mathbf{a}\times]\mathbf{b} = \mathbf{a}\times\mathbf{b}$. If $\boldsymbol{\tau}^*$ is the time-optimal torque history and $\boldsymbol{\omega}^*$, $\boldsymbol{\lambda}^*$ represent the solutions of Eqs. (1) and (5) for $\boldsymbol{\tau} = \boldsymbol{\tau}^*$, then according to *Pontryagin's minimum principle*, optimal torque $\boldsymbol{\tau}^*$ satisfies the equation

$$H(\boldsymbol{\omega}^*, \boldsymbol{\lambda}^*, \boldsymbol{\tau}^*) \le H(\boldsymbol{\omega}^*, \boldsymbol{\lambda}^*, \boldsymbol{\tau}), \quad \forall\ \boldsymbol{\tau} \in \mathbb{R}^3 \ni \|\boldsymbol{\tau}\| \le \tau_{\max} \tag{7}$$

for every $t \in [0, t_f)$. Equations (4) and (7) together imply that

$$\boldsymbol{\tau}^* = -\frac{\mathbf{I}_c^{-1}\boldsymbol{\lambda}^*}{\|\mathbf{I}_c^{-1}\boldsymbol{\lambda}^*\|}\tau_{\max} \tag{8}$$

Therefore, the dynamics of the closed-loop system becomes

$$\dot{\boldsymbol{\omega}}^* = \boldsymbol{\phi}(\boldsymbol{\omega}^*) - \frac{\mathbf{I}_c^{-2}\boldsymbol{\lambda}^*}{\|\mathbf{I}_c^{-1}\boldsymbol{\lambda}^*\|}\tau_{\max} \tag{9}$$

The structure of the optimal controller is determined by Eqs. (5) and (8) together. However, to determine the control input the initial values of the costates, $\boldsymbol{\lambda}(0)$, should be also obtained. In fact, by choosing different initial values for the costates, we obtain a family of optimal solutions, each of which corresponds to a particular final angular velocity. In general, the two-point boundary value problem for nonlinear systems is challenging. However, as it will be shown in the following, the structure of our particular system (5) and (9) leads to an easy solution when the final velocity is zero. In such a case, it will be shown that the costates and states are related via the following function:

$$\boldsymbol{\lambda}^*(t) = \frac{\mathbf{I}_c^2\boldsymbol{\omega}^*}{\|\mathbf{I}_c\boldsymbol{\omega}^*\|\tau_{\max}} \quad \forall\ t \in [0, t_f) \tag{10}$$

despite the fact that the evolutions of the optimal trajectories of the states and costates are governed by two different differential equations (5) and (9). In other words, Eq. (10) is a solution to Eqs. (5) and (9). Note that since $\boldsymbol{\omega}^*(t) = \boldsymbol{\omega}(t) \quad \forall\ t \in [0, t_f)$, it is not obvious that $\boldsymbol{\omega}^*(t_f)$ will be zeros. On the substitution of Eq. (10) into Eq. (9), we arrive at the following autonomous system:

$$\dot{\boldsymbol{\omega}}^* = \boldsymbol{\phi}(\boldsymbol{\omega}^*) - \frac{\boldsymbol{\omega}^*}{\|\mathbf{I}_c\boldsymbol{\omega}^*\|}\tau_{\max} \quad \forall\ t \in [0, t_f) \tag{11}$$

To prove that Eq. (10) represents the optimal trajectory, we need to show that Eqs. (10) and (11) satisfy the optimality condition (5). Using Eq. (11) in the time derivative of the right-hand side (RHS) of Eq. (10) yields

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\lambda}^* = \mathbf{I}_c^2\frac{\boldsymbol{\phi} - \frac{\boldsymbol{\omega}^*}{\|\mathbf{I}_c\boldsymbol{\omega}^*\|}\tau_{\max}}{\|\mathbf{I}_c\boldsymbol{\omega}^*\|\tau_{\max}} - \mathbf{I}_c^2\boldsymbol{\omega}^*\frac{\boldsymbol{\omega}^{*T}\mathbf{I}_c^2(\boldsymbol{\phi} - \frac{\boldsymbol{\omega}^*}{\|\mathbf{I}_c\boldsymbol{\omega}^*\|}\tau_{\max})}{\|\mathbf{I}_c\boldsymbol{\omega}^*\|^3\tau_{\max}} = \frac{\mathbf{I}_c^2\boldsymbol{\phi}}{\|\mathbf{I}_c\boldsymbol{\omega}^*\|\tau_{\max}} \tag{12}$$

On the other hand, using Eqs. (6) and (10) in the RHS of Eq. (5) yields

$$-\frac{\partial \boldsymbol{\phi}^T}{\partial \boldsymbol{\omega}}\boldsymbol{\lambda}^* = \frac{-\mathbf{I}_c[\boldsymbol{\omega}^*\times]\mathbf{I}_c\boldsymbol{\omega}^* + (\mathbf{I}_c\boldsymbol{\omega}^*)\times\mathbf{I}_c\boldsymbol{\omega}^*}{\|\mathbf{I}_c\boldsymbol{\omega}^*\|\tau_{\max}} = \frac{\mathbf{I}_c^2\boldsymbol{\phi}}{\|\mathbf{I}_c\boldsymbol{\omega}^*\|\tau_{\max}} \tag{13}$$

A comparison between Eqs. (12) and (13) clearly proves that Eq. (10) is indeed a solution to the differential equation (5). Furthermore, the Hamiltonian on the optimal trajectory becomes

$$H^* = \boldsymbol{\lambda}^{*T}\boldsymbol{\phi}(\boldsymbol{\omega}^*) = -\frac{(\mathbf{I}_c\boldsymbol{\omega}^*)^T[\boldsymbol{\omega}^*\times](\mathbf{I}_c\boldsymbol{\omega}^*)}{\|\mathbf{I}_c\boldsymbol{\omega}^*\|\tau_{\max}} = 0$$

Therefore, the condition for optimality with open end time is also satisfied [22]. The substitution of Eq. (10) into Eq. (8) gives

$$\boldsymbol{\tau}^* = -\frac{\mathbf{I}_c\boldsymbol{\omega}^*}{\|\mathbf{I}_c\boldsymbol{\omega}^*\|}\tau_{\max} \quad \forall\ t \in [0, t_f) \tag{14}$$

Apparently, the control law (14) constitutes a nonlinear state feedback. The structure of Eq. (14) also gives an interesting insight into the optimal control solution: the vectors of instantaneous torque and the angular momentum are parallel but in the opposite direction.

Clearly, the torque controller should be turned off right after time t_f because the optimal solution is valid only for the time interval $[0 t_f)$. However, t_f is not given beforehand; rather it is one of the arguments of the optimization process. To be able to obtain the terminal time, let us define h as the magnitude of the angular momentum, that is,

$$h \triangleq \|\mathbf{I}_c\omega\| \tag{15}$$

The time derivative of h along the optimal trajectory (9) satisfies

$$\dot{h} = \frac{\boldsymbol{\omega}^T\mathbf{I}_c^2\dot{\boldsymbol{\omega}}}{\|\mathbf{I}_c\boldsymbol{\omega}\|} = -\frac{(\mathbf{I}_c\boldsymbol{\omega})^T[\boldsymbol{\omega}\times](\mathbf{I}_c\boldsymbol{\omega})}{\|\mathbf{I}_c\boldsymbol{\omega}\|} - \frac{(\boldsymbol{\omega}^T\mathbf{I}_c^2\boldsymbol{\omega})\tau_{\max}}{\|\mathbf{I}_c\boldsymbol{\omega}\|^2} = -\tau_{\max}$$

This means that the time-optimal controller reduces the magnitude of the angular momentum linearly at the constant rate of $\tau_{\max}$. Therefore, given initial angular velocity $\boldsymbol{\omega}(0)$, the terminal time can be computed from

$$t_f = \frac{\|\mathbf{I}_c\boldsymbol{\omega}(0)\|}{\tau_{\max}} \tag{16}$$

Alternatively, the torque feedback can be turned off when the angular velocity becomes sufficiently small. Assume that the terminal condition is stated by

$$\|\boldsymbol{\omega}(t_f)\| \le \epsilon \tag{17}$$

with ϵ being selected to be arbitrary small. Then, the optimal torque feedback can be specified by

$$\boldsymbol{\tau} = \begin{cases} -\frac{\mathbf{I}_c\boldsymbol{\omega}}{\|\mathbf{I}_c\boldsymbol{\omega}\|}\tau_{\max} & \text{if } \|\boldsymbol{\omega}\| \ge \epsilon \\ \mathbf{0} & \text{otherwise} \end{cases} \tag{18}$$

Now, it remains to show that the terminal condition (17) is satisfied under control law (18). To this end, define the following Lyapunov function:

$$V(\boldsymbol{\omega}) = \tfrac{1}{2}\boldsymbol{\omega}^T\mathbf{I}_c\boldsymbol{\omega}$$

From the identity $\boldsymbol{\omega}^T\mathbf{I}_c\boldsymbol{\phi}(\boldsymbol{\omega}) = 0$, the time derivative of the above function along trajectories of the closed-loop system (11) is obtained to be seminegative definite, that is,

$$\dot{V} = \begin{cases} -\frac{V}{\|\mathbf{I}_c\boldsymbol{\omega}\|}\tau_{\max} & \text{if } \|\boldsymbol{\omega}\| \ge \epsilon \\ 0 & \text{otherwise} \end{cases} \le 0$$

Therefore, according to LaSalle's global invariant set theorem [23,24], the state trajectory must converge to the invariant set $\Omega \subseteq \{\boldsymbol{\omega}\colon \|\boldsymbol{\omega}\| < \epsilon\}$. This means that the optimal controller (18) reduces the tumbling rate to the residual error ϵ.

B. Nonzero Final Velocity

A spin-stabilized spacecraft is spun up around a single eigenaxis to stabilize the orientation of that axis, using the gyroscopic effect. To drive a tumbling spacecraft to a final spin-stabilized state, the time-optimal controller is extended for the case where the desired final velocity is nonzero but it is parallel to the eigenaxis. Since all spin-stabilized spacecraft are designed to be axially symmetric [25], we assume that the inertia of the spacecraft is axisymmetric relative to the z axis, that is, the principle moment of inertia is $\mathbf{I}_c = \operatorname{diag}\{I_{xx}, I_{xx}, I_{zz}\}$. The condition for the final velocity $\boldsymbol{\omega}_f = \boldsymbol{\omega}(t_f) \neq \mathbf{0}$ to be a spin-stabilized velocity is that it is parallel to the axisymmetric axis, that is,

$$\boldsymbol{\omega}_f = \begin{bmatrix} 0 \\ 0 \\ \omega_f \end{bmatrix} \tag{19}$$

Now, consider the following change of variable:

$$\bar{\boldsymbol{\omega}} = \boldsymbol{\omega} - \boldsymbol{\omega}_f \quad \text{where } \bar{\boldsymbol{\omega}}(t_f) = \mathbf{0} \tag{20}$$

Then, the Euler's equation in terms of the new variable can be written as

$$\begin{aligned} \dot{\bar{\boldsymbol{\omega}}} &= \boldsymbol{\phi}(\bar{\boldsymbol{\omega}} + \boldsymbol{\omega}_f) + \mathbf{I}_c^{-1}\boldsymbol{\tau} = \boldsymbol{\phi}(\bar{\boldsymbol{\omega}}) + \mathbf{I}_c^{-1}(\boldsymbol{\omega}_f \times \mathbf{I}_c\boldsymbol{\omega}_f + \boldsymbol{\omega}_f \\ &\quad \times \mathbf{I}_c\bar{\boldsymbol{\omega}} + \bar{\boldsymbol{\omega}} \times \mathbf{I}_c\boldsymbol{\omega}_f) + \mathbf{I}_c^{-1}\boldsymbol{\tau} = \boldsymbol{\phi}(\bar{\boldsymbol{\omega}}) + \mathbf{M}\bar{\boldsymbol{\omega}} + \mathbf{I}_c^{-1}\boldsymbol{\tau} \end{aligned} \tag{21}$$

where $\mathbf{M}$ is a constant skew-symmetric matrix, which can be constructed from the values of the inertia matrix and the final angular rate as

$$\mathbf{M} \triangleq \begin{bmatrix} 0 & -\frac{I_{xx}-I_{zz}}{I_{xx}}\omega_f & 0 \\ \frac{I_{xx}-I_{zz}}{I_{xx}}\omega_f & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{22}$$

Here, note that since $\boldsymbol{\omega}_f$ is specified to be parallel to the eigenaxis, then $\boldsymbol{\omega}_f \times \mathbf{I}_c\boldsymbol{\omega}_f = \mathbf{0}$. It follows the system Hamiltonian as

$$H = 1 + \boldsymbol{\lambda}^T\boldsymbol{\phi}(\bar{\boldsymbol{\omega}}) + \boldsymbol{\lambda}^T\mathbf{M}\bar{\boldsymbol{\omega}} + (\mathbf{I}_c^{-1}\boldsymbol{\lambda})^T\boldsymbol{\tau} \tag{23}$$

Clearly, the torque restriction (3) implies a similar structure as Eq. (8). Therefore, in a development similar to Eqs. (11–13), one can show that the optimal costates and states are related by

$$\boldsymbol{\lambda}^*(t) = \frac{\mathbf{I}_c^2\bar{\boldsymbol{\omega}}^*}{\|\mathbf{I}_c\bar{\boldsymbol{\omega}}^*\|\tau_{\max}} \quad \forall\ t \in [0, t_f)$$

which, in turn, gives the optimal torque as

$$\boldsymbol{\tau}^* = -\frac{\mathbf{I}_c(\boldsymbol{\omega} - \boldsymbol{\omega}_f)}{\|\mathbf{I}_c(\boldsymbol{\omega} - \boldsymbol{\omega}_f)\|}\tau_{\max} \tag{24}$$

By inspection, one can show that

$$\boldsymbol{\lambda}^{*T}\mathbf{M}\bar{\boldsymbol{\omega}} \equiv 0 \tag{25}$$

Using the above identity in Eq. (23) yields

$$H^* = 1 + \boldsymbol{\lambda}^{*T}\boldsymbol{\phi}(\bar{\boldsymbol{\omega}}^*) + (\mathbf{I}_c^{-1}\boldsymbol{\lambda})^T\boldsymbol{\tau}^* = 0$$

which is similar to the Hamiltonian of the zero final velocity system (4). Therefore, using the same argument as in the previous section, one can conclude that Eq. (24) is the time-optimal controller satisfying the Euclidean norm bound on the torque and the terminal condition $\bar{\boldsymbol{\omega}}(t_f) = \mathbf{0}$.

Similarly, the terminal time for the case of nonzero final velocity can be determined. Analogous to Eq. (15), let us define variable

$$\bar{h} = \|\mathbf{I}_c\bar{\boldsymbol{\omega}}\|$$

whose time derivative satisfies

$$\dot{\bar{h}} = -\tau_{\max} + \frac{\bar{\boldsymbol{\omega}}^T(\mathbf{I}_c\mathbf{M})\bar{\boldsymbol{\omega}}}{\|\mathbf{I}_c\bar{\boldsymbol{\omega}}\|} = -\tau_{\max} \tag{26}$$

where the second term on the RHS of Eq. (26) vanishes because $\mathbf{I}_c\mathbf{M}$ is a skew-symmetric matrix. Then, since $\bar{\boldsymbol{\omega}}(0) = \boldsymbol{\omega}(0) - \boldsymbol{\omega}_f$ and $\bar{\boldsymbol{\omega}}(t_f) = \mathbf{0}$, the terminal time can be calculated from

$$t_f = \frac{\|\ \mathbf{I}_c(\boldsymbol{\omega}(0) - \boldsymbol{\omega}_f)\ \|}{\tau_{\max}} \tag{27}$$

As illustrated in Fig. 1, the proposed time-optimal controller can be easily implemented.

III. Examples

A. Passivation of a Tumbling Spacecraft

Consider a spacecraft with inertial tensor

$$\mathbf{I}_c = \begin{bmatrix} 1 & 0.5 & -1 \\ 0.5 & 2 & 1 \\ -1 & 1 & 5 \end{bmatrix} \text{kg}\cdot\text{m}^2 \tag{28}$$

that tumbles with angular rate

$$\boldsymbol{\omega}(0) = \begin{bmatrix} 1.0 \\ 2.0 \\ 0.5 \end{bmatrix} \text{rad/s} \tag{29}$$

at time $t = 0$ s. The objective is to bring the tumbling spacecraft to rest in minimum time while the magnitude of the torque that can be applied to the body is restricted by

$$\|\boldsymbol{\tau}\| \leq 0.5 \text{ Nm} \tag{30}$$

According to Eq. (16), given the initial angular momentum of $h(0) = 9.67\ \text{kg}\cdot\text{m}^2/\text{s}$, the optimal controller is expected to achieve complete passivation within $t_f = 19.34$ s. Figure 2 illustrates the trajectories of the optimal torque applied to the satellite according to the control law (18). The subsequent trajectories of the angular velocities are shown in Fig. 3. Apparently, the controller succeeded to reduce the angular velocity of the rigid body to zero within the expected finite time t_f. As shown in Fig. 4, the magnitude of the angular momentum decays linearly to zero from its initial values.

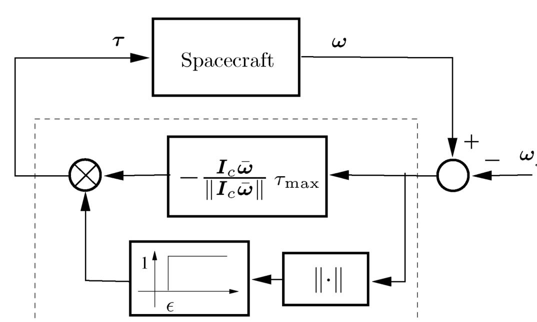

Fig. 1 The time-optimal detumbling controller.

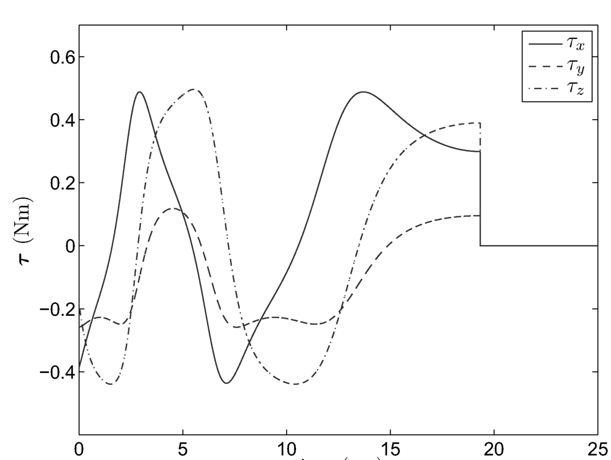

Fig. 2 Optimal torque trajectories for $\boldsymbol{\omega}_f = \mathbf{0}$.

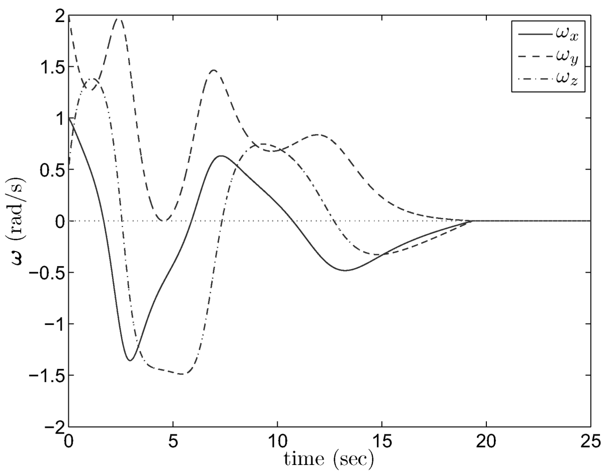

Fig. 3 Trajectories of the angular velocities for $\omega_f = 0$.

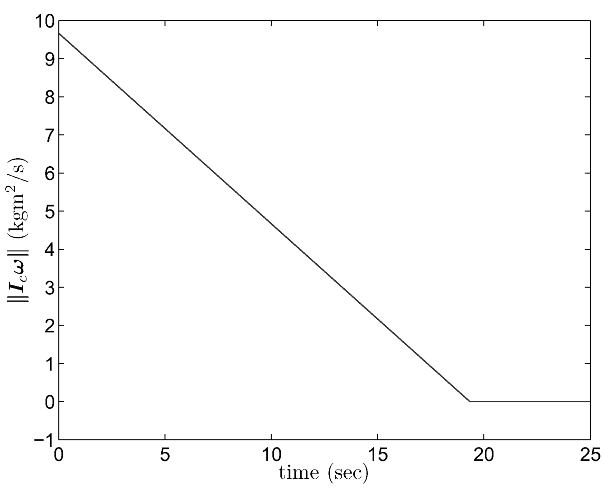

Fig. 4 The magnitude of angular momentum.

B. Spin-Stabilized Spacecraft

This example illustrates the application of the time-optimal controller to bring an axisymmetric spacecraft from its initial tumbling motion into a spin-stabilized motion. The spacecraft is with principal moment of inertia $\mathbf{I}_c = \text{diag}\{4, 4, 2\}\ \text{kg} \cdot \text{m}^2$ and the initial angular rate as specified in Eq. (31). The objective is that the spacecraft is spun up along its symmetric axis with final angular rate

$$\boldsymbol{\omega}_f = \begin{bmatrix} 0.0 \\ 0.0 \\ 2.0 \end{bmatrix} \text{rad/s} \tag{31}$$

in minimum time, whereas the actuation torque is still limited by Eq. (30). According to Eq. (27), given the difference between the initial and final angular velocities, the optimal maneuver to bring the tumbling spacecraft to the spin-stabilized state takes $t_f = 18.86$ s. Figures 5 and 6 illustrate the torque and angular rate trajectories of the time-optimal maneuvers. As shown in Fig. 6, the spacecraft reaches the desired spin-stabilized state within the predicted terminal time. It should be pointed out that because a spinning rigid body about its eigenaxis constitutes a stable equilibrium point, the spacecraft can sustain its angular rate $\boldsymbol{\omega}_f$ after the terminal time even if the controller is turned off.

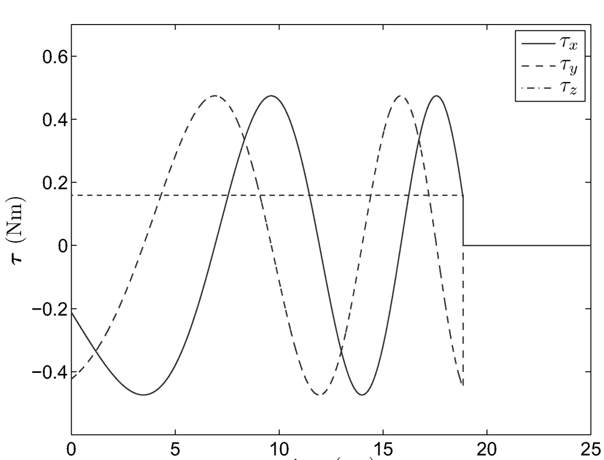

Fig. 5 Optimal torque trajectories for the nonzero final velocity.

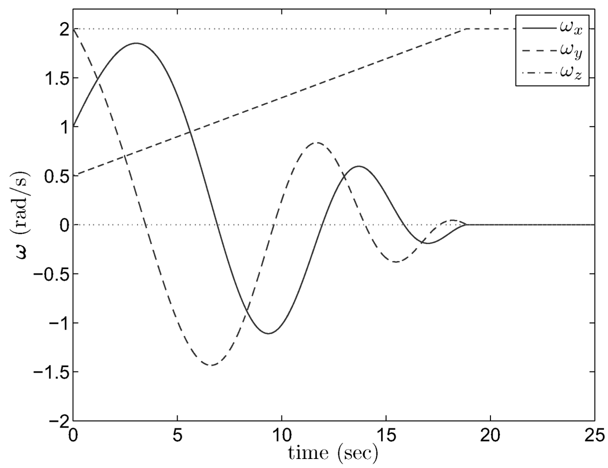

Fig. 6 Trajectories of the angular velocities to the final spin-stabilized state.

IV. Conclusions

A closed-form solution to the time-optimal control problem relating to the detumbling maneuvers of a spacecraft is presented in a finite-time interval subject to the constraint that the magnitude of the driving torques is below a prescribed value. It has been shown that the system costate is explicitly related to the state vector by a nonlinear static function. This relationship has drastically simplified the optimal torque solution, which is then given in the form of a nonlinear state feedback. It turned out that the optimal torque solution has a nice physical interpretation: the vectors of the instantaneous torque and the angular momentum are parallel but in the opposite direction. It has also been shown that the optimal controller decreases the magnitude of the angular momentum linearly by time at the constant rate of $\tau_{\max}$. This fact allowed the simple derivation of the final time, which could be then used to turn off the feedback when the time interval has elapsed. Alternatively, it has been shown that the optimal torque feedback for detumbling maneuvers could be turned off when the tumbling rate is reduced to a residual error. Finally, the time-optimal control technique has been extended for the case wherein the final velocity is nonzero but it is parallel to the eigenaxis. The optimal controller can be applied to drive a tumbling axisymmetric spacecraft to a final spin-stabilized state in minimum time given the limitation of an actuator torque.

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